

Processes $e^+e^- \rightarrow \pi^0(\pi^{0'})\gamma$ in the NJL model

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Abstract The processes of electron-positron annihilation into $\pi^0\gamma$ and into $\pi'(1300)\gamma$ are considered within the NJL model. Intermediate vector mesons ρ^0 , ω , $\rho'(1450)$, and $\omega'(1420)$ are taken into account. The latter two mesons are treated as the first radial excited states. They are incorporated into the NJL model by means of a polynomial form factor. Numerical predictions for the cross sections of these processes are received for the center-of-mass energies below 2 GeV. Our results for the $\pi^0\gamma$ production are in agreement with experimental data received in the energy region 600 – 1020 MeV.

PACS. 12.39.Fe Chiral Lagrangians 13.20.Jf Decays of other mesons 13.66.Bc Hadron production in e-e+ interactions

1 Introduction

In our recent work [1] the process $e^+e^- \rightarrow \pi^0\omega$ was considered in the framework of the extended Nambu–Jona-Lasinio (NJL) model [2,3,4]. In that work we took into account $\rho(770)$ and $\rho'(1450)$ intermediate vector meson states. The obtained results were found to be in a satisfactory agreement with the experimental data [5] at energies below 2 GeV. Our results are also in a qualitative agreement with the phenomenological description of this process received within a vector meson dominance model [5] by fitting several free parameters from the experimental data. It is worth to note that in our approach the process was described without introduction of any additional arbitrary parameter. This testifies about the predictive power of the NJL model and gives us a hope to obtain reliable predictions for other similar processes in the same energy range.

In the present paper we consider the processes $e^+e^- \rightarrow \pi^0(\pi^{0'})\gamma$ which have small cross sections because they are suppressed by factor α with respect to other typical channels of electron-positron annihilation into hadrons. Nevertheless, the process $e^+e^- \rightarrow \pi^0\gamma$ was studied with a rather high accuracy in the energy regions around the ω and ϕ meson masses [6,7,8]. The ongoing high-luminosity experiments in Novosibirsk (VEPP-2000) and Beijing (BES-III) will also collect considerable statistics for many possible annihilation processes including the ones with production of $\pi'(1300)$. So theoretical predictions for the given processes should be of interest for the physical programs of these colliders.

2 Lagrangian and process amplitudes

For the description of the first three diagrams (with intermediate γ , ρ , and ω states) for the process of $\pi^0\gamma$ production, see Figs. 1 and 2, we need the part of the standard NJL Lagrangian which describes interactions of photons, pions, and the ground states of vector mesons with quarks, see refs. [9,10,11]. It has the form

$$\Delta\mathcal{L}_1 = \bar{q} \left[i\hat{\partial} - m + \frac{e}{2} \left(\tau_3 + \frac{1}{3}I \right) \hat{A} + ig_\pi \gamma_5 \tau_3 \pi^0 + \frac{g_\rho}{2} \gamma_\mu (I\hat{\omega} + \tau_3 \hat{\rho}^0) \right] q, \quad (1)$$

where $\bar{q} = (\bar{u}, \bar{d})$ with u and d quark fields; $m = \text{diag}(m_u, m_d)$, $m_u = m_d = 280$ MeV is the constituent quark mass; e is the electron charge; A , π^0 , ω and ρ^0 are the photon, pion, ω and ρ meson fields, respectively; g_π is the pion coupling constant, $g_\pi = m_u/f_\pi$, where $f_\pi = 93$ MeV is the pion decay constant; g_ρ is the vector meson coupling constant, $g_\rho \approx 6.14$ corresponding to the standard relation $g_\rho^2/(4\pi) \approx 3$; $I = \text{diag}(1, 1)$ and τ_3 is the third Pauli matrix.

For description of the radial excited mesons interactions we use the extended version of the NJL Lagrangian [2, 3, 12]:

$$\Delta\mathcal{L}_2^{\text{int}} = \bar{q}(k') \left\{ A_\pi \tau^3 \gamma_5 \pi^0(p) - A_{\pi'} \tau^3 \gamma_5 \pi'(p) + A_{\omega,\rho} (\tau^3 \hat{\rho}(p) + I\hat{\omega}(p)) - A_{\omega',\rho'} (\tau^3 \hat{\rho}'(p) + I\hat{\omega}'(p)) \right\} q(k), \quad p = k - k', \quad (2)$$

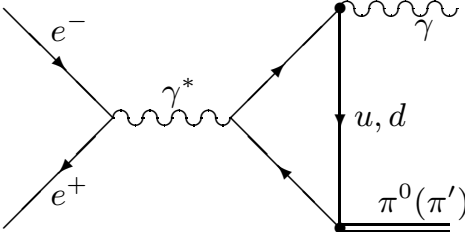


Figure 1. The Feynman diagram with photon exchange.

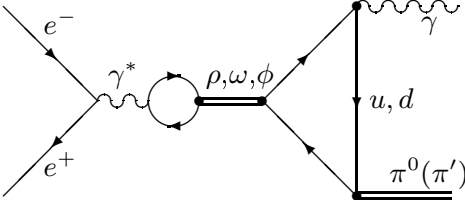


Figure 2. Feynman diagram(s) with ρ^0 , ω and ϕ meson exchange.

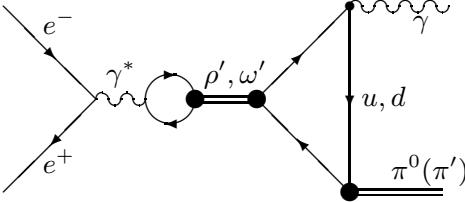


Figure 3. Feynman diagram(s) with ρ' and ω' meson exchange.

$$\begin{aligned}
 A_\pi &= g_{\pi_1} \frac{\sin(\alpha + \alpha_0)}{\sin(2\alpha_0)} + g_{\pi_2} f(k^\perp{}^2) \frac{\sin(\alpha - \alpha_0)}{\sin(2\alpha_0)}, \\
 A_{\pi'} &= g_{\pi_1} \frac{\cos(\alpha + \alpha_0)}{\sin(2\alpha_0)} + g_{\pi_2} f(k^\perp{}^2) \frac{\cos(\alpha - \alpha_0)}{\sin(2\alpha_0)}, \\
 A_{\omega, \rho} &= g_{\rho_1} \frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} + g_{\rho_2} f(k^\perp{}^2) \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)}, \\
 A_{\omega', \rho'} &= g_{\rho_1} \frac{\cos(\beta + \beta_0)}{\sin(2\beta_0)} + g_{\rho_2} f(k^\perp{}^2) \frac{\cos(\beta - \beta_0)}{\sin(2\beta_0)}.
 \end{aligned}$$

The radially-excited states were introduced in the NJL model with the help of the form factor in the quark-meson interaction:

$$\begin{aligned}
 f(k^\perp{}^2) &= (1 - d|k^\perp{}^2|)\Theta(\Lambda^2 - |k^\perp{}^2|), \\
 k^\perp &= k - \frac{(kp)p}{p^2}, \quad d = 1.78 \text{ GeV}^{-2}, \quad (3)
 \end{aligned}$$

where k and p are the quark and meson momenta, respectively. The cut-off parameter $\Lambda = 1.03 \text{ GeV}$ is taken [13]. The filled circles in fig. 3 denote the presence of the form factor in the quark-meson vertexes. Coupling constants $g_{\pi_1} = g_\pi$ and $g_{\rho_1} = g_\rho$ are the same as in the standard NJL version. Constants $g_{\pi_2} = 3.20$, $g_{\rho_2} = 9.87$, and the mixing angles $\alpha_0 = 59.06^\circ$, $\alpha = 59.38^\circ$, $\beta_0 = 61.53^\circ$, and $\beta = 76.78^\circ$ were defined in refs. [12,14].

A. The process $e^+ + e^- \rightarrow \pi^0 + \gamma$ contains contributions of three amplitudes:

$$T^\lambda = \bar{e} \gamma_\mu e \varepsilon_{\mu\lambda\alpha\beta} \frac{p_\pi^\alpha p_\gamma^\beta}{ms} \{B_\gamma + B_{\rho+\omega+\phi} + B_{\rho'+\omega'}\}, \quad (4)$$

where $s = (p_1(e^+) + p_2(e^-))^2$. Contribution $A_\gamma^{\mu\lambda}$ corresponds to triangular diagram in fig. 1, *i.e.* the pion transition form factor:

$$B_\gamma = 2V_{\gamma^*\pi^0\gamma}(s). \quad (5)$$

The sum of ρ and ω meson contributions (see fig. 2) reads

$$\begin{aligned}
 B_{\rho+\omega+\phi} &= \left\{ \frac{s}{s - M_\rho^2 + iM_\rho\Gamma_\rho} + \frac{s}{s - M_\omega^2 + iM_\omega\Gamma_\omega} \right. \\
 &\quad \left. + \frac{s\sqrt{2}\sin\theta_{\omega\phi}}{s - M_\phi^2 + iM_\phi\Gamma_\phi} \right\} \frac{1}{g_{\rho_1}} V_{\rho\pi^0\gamma}(s), \quad (6)
 \end{aligned}$$

where $\gamma - \rho(\omega, \phi)$ transitions via quark loops [9] are taken into account. Note that in the case of ω meson the relative factor $1/3$ in the $\gamma - \omega$ transition (with respect to the ρ meson case) is canceled out with factor 3 in the $\omega\pi^0\gamma$ vertex. Factor $\sqrt{2}$ in the numerator of the ϕ meson propagator arise from the $\gamma - \phi$ through the s quark loop. The standard value of the $\phi - \omega$ mixing angle $\theta_{\omega\phi} \approx -3^\circ$ is used [10,11,15].

Let us emphasize that the sum of the photon, ρ , and ω meson contributions gives the expression coinciding with the result of vector meson dominance (VMD) model, which emerges from the NJL model, see also [9,17,18,19].

The contributions of the excited mesons are calculated in an analogous way but with the extended Lagrangian (2). We get

$$\begin{aligned}
 B_{\rho'+\omega'} &= \left(-\frac{\cos(\beta + \beta_0)}{\sin(2\beta_0)} - \Gamma \frac{\cos(\beta - \beta_0)}{\sin(2\beta_0)} \right) \frac{1}{g_{\rho_1}} \\
 &\quad \times \left\{ \frac{s}{s - M_{\rho'}^2 + iM_{\rho'}\Gamma_{\rho'}} + \frac{s}{s - M_{\omega'}^2 + iM_{\omega'}\Gamma_{\omega'}} \right\} \\
 &\quad \times V_{\rho'\pi^0\gamma}(s). \quad (7)
 \end{aligned}$$

We checked that the possible effect of taking into account the running of the meson widths is small. So for the numerical calculations we use the values from the Particle Data Group [16]: $\Gamma_\rho = 146.2 \text{ MeV}$, $\Gamma_\omega = 8.49 \text{ MeV}$, $\Gamma_{\rho'} = 400 \text{ MeV}$, and $\Gamma_{\omega'} = 215 \text{ MeV}$.

The vertexes are defined via the triangular loop integrals of the anomalous type:

$$\begin{aligned}
 V_{\gamma^*\pi^0\gamma} &= g_{\pi_1} I_0^{(3)}, \\
 I_n^{(3)} &= - \int \frac{d^4k}{i\pi^2} \frac{m^2 f^n(k^\perp{}^2) \Theta(\Lambda^2 - |k^\perp{}^2|)}{(k^2 - m^2 + i0)} \\
 &\quad \times \frac{1}{((k + p_\gamma)^2 - m^2 + i0)((k - p_\pi)^2 - m^2 + i0)}, \quad (8)
 \end{aligned}$$

$$\begin{aligned}
 V_{\rho\pi^0\gamma} &= g_{\pi_1} \left(\frac{\sin(\beta + \beta_0)g_{\rho_1}I_0^{(3)}}{\sin(2\beta_0)} + \frac{\sin(\beta - \beta_0)g_{\rho_2}I_1^{(3)}}{\sin(2\beta_0)} \right), \\
 V_{\rho'\pi^0\gamma} &= -g_{\pi_1} \left(\frac{\cos(\beta + \beta_0)g_{\rho_1}I_0^{(3)}}{\sin(2\beta_0)} + \frac{\cos(\beta - \beta_0)g_{\rho_2}I_1^{(3)}}{\sin(2\beta_0)} \right),
 \end{aligned}$$

The vertex factors for ω mesons $V_{\omega\pi^0\gamma}$ and $V_{\omega'\pi^0\gamma}$ are just three times greater than the ρ meson ones. In the p^2 approximation the triangular diagrams reproduce the $<$ Wess-Zumino terms in the chiral symmetric meson Lagrangian [9,10,11,20]. With help of these terms one can describe radiative decays of all pseudoscalar and vector meson nonets in a satisfactory agreement with experimental data [9,10,11]. In this work we will use the same approximation¹.

The transition of photon into ρ meson reads [9]:

$$\frac{e}{g_\rho}(g^{\nu\nu'}q^2 - q^\nu q^{\nu'}). \quad (9)$$

The $\gamma - \omega$ transition differs from the above just by factor $1/3$. In the amplitudes with excited mesons, we have to take into account the $\gamma - \rho_2$ and $\gamma - \omega_2$ transitions ($\gamma - \rho_1(\omega_1)$ ones are the same as in the standard $\gamma - \rho(\omega)$ cases) can be expressed via the $\gamma - \rho(\omega)$ transition with the additional factor [3,12]

$$\Gamma = \frac{I_2^f}{\sqrt{I_2 I_2^f}} \approx 0.47. \quad (10)$$

In particular, the $\gamma - \rho'$ transition takes the form

$$\frac{e}{g_\rho}(g^{\nu\nu'}q^2 - q^\nu q^{\nu'}) \left\{ \frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} + \Gamma \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} \right\}.$$

Note that the relative factors $1/3$ (in photon-meson transitions) and 3 (in vertexes) in contributions of ρ and ω mesons cancel each other.

B. The process $e^+ + e^- \rightarrow \pi'(1300) + \gamma$ can be described in a very similar manner to the previous one. The main difference is in the vertexes $V_{\gamma^*(\rho,\rho',\omega,\omega')\pi'\gamma}$. They read

$$\begin{aligned} V_{\gamma^*\pi'\gamma} &= V_{\gamma^*\pi^0\gamma} \frac{\cos(\alpha + \alpha_0)}{\sin(2\alpha_0)} + g_{\pi_2} I_1^{(3)} \frac{\cos(\alpha - \alpha_0)}{\sin(2\alpha_0)}, \\ V_{\rho\pi'\gamma} &= -g_{\rho_1} \frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} g_{\pi_1} \frac{\cos(\alpha + \alpha_0)}{\sin(2\alpha_0)} I_0^{(3)} \\ &\quad - g_{\rho_2} \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} g_{\pi_1} \frac{\cos(\alpha + \alpha_0)}{\sin(2\alpha_0)} I_1^{(3)} \\ &\quad - g_{\rho_1} \frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} g_{\pi_2} \frac{\cos(\alpha - \alpha_0)}{\sin(2\alpha_0)} I_1^{(3)} \\ &\quad - g_{\rho_2} \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} g_{\pi_2} \frac{\cos(\alpha - \alpha_0)}{\sin(2\alpha_0)} I_2^{(3)}, \\ V_{\rho'\pi'\gamma} &= g_{\rho_1} \frac{\cos(\beta + \beta_0)}{\sin(2\beta_0)} g_{\pi_1} \frac{\cos(\alpha + \alpha_0)}{\sin(2\alpha_0)} I_0^{(3)} \\ &\quad + g_{\rho_2} \frac{\cos(\beta - \beta_0)}{\sin(2\beta_0)} g_{\pi_1} \frac{\cos(\alpha + \alpha_0)}{\sin(2\alpha_0)} I_1^{(3)} \\ &\quad + g_{\rho_1} \frac{\cos(\beta + \beta_0)}{\sin(2\beta_0)} g_{\pi_2} \frac{\cos(\alpha - \alpha_0)}{\sin(2\alpha_0)} I_1^{(3)} \\ &\quad + g_{\rho_2} \frac{\cos(\beta - \beta_0)}{\sin(2\beta_0)} g_{\pi_2} \frac{\cos(\alpha - \alpha_0)}{\sin(2\alpha_0)} I_2^{(3)}. \end{aligned} \quad (11)$$

¹ The cut-off in the loop integral (9) will be used only in the presence of the form factor.

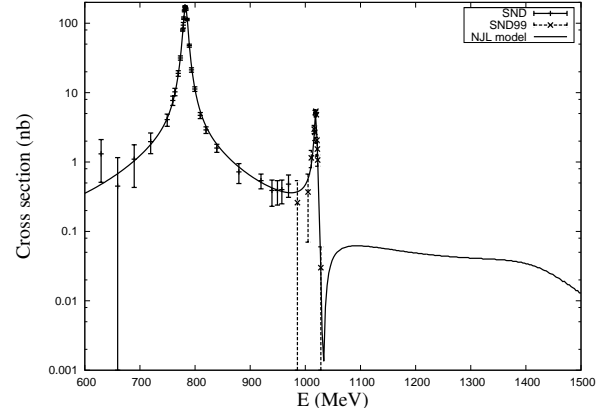


Figure 4. Comparison of experimental results for $e^+e^- \rightarrow \pi^0\gamma$ with the NJL model prediction.

All other factors are the same as in the pion production case. But the propagators of ρ' and ω' mesons can be taken with constant widths: $\Gamma_{\rho'} = 340$ MeV and $\Gamma_{\omega'} = 215$ MeV.

Note that within the same approach the listed here vertexes allow us to get a good description the radiative decays of meson ground states: $\Gamma_{\pi^0 \rightarrow \gamma\gamma} = 7.7$ eV, $\Gamma_{\rho^0 \rightarrow \pi^0\gamma} = 77$ keV, $\Gamma_{\omega \rightarrow \pi^0\gamma} = 710$ keV. The corresponding experimental values [16] are $\Gamma_{\pi^0 \rightarrow \gamma\gamma}^{\text{exp}} = 7.8 \pm 0.5$ eV, $\Gamma_{\rho^0 \rightarrow \pi^0\gamma}^{\text{exp}} = 88 \pm 12$ keV, $\Gamma_{\omega \rightarrow \pi^0\gamma}^{\text{exp}} = 700 \pm 30$ keV.

3 Cross section and Numerical Results

Now we can estimate the contributions of the considered amplitudes into the total process cross section. The details of phase volume calculations and evaluation of the cross section can be found in ref. [21]. For our case it takes the form

$$\begin{aligned} \sigma^{e^+e^- \rightarrow \pi\gamma}(s) &= \frac{\alpha^3}{24\pi^2 s^3 f_\pi^2} \lambda^{3/2}(s, M_\omega^2, M_\pi^2) \frac{1}{g_{\pi_1}^2} \\ &\quad \times |B_\gamma + B_{\rho+\omega+\phi} + B_{\rho'+\omega'}|^2 \\ \lambda(s, M_\omega^2, M_\pi^2) &= (s - M_\omega^2 - M_\pi^2)^2 - 4M_\omega^2 M_\pi^2. \end{aligned} \quad (12)$$

The cross section for $\pi'\gamma$ production has the same form with simple substitutions.

Fig. 4 shows the experimental data of the SND collaboration [7,22] and the corresponding theoretical result (the solid line) received within the applied here NJL phenomenological model. The NJL model predictions are in a good agreement with the experimental data. In particular, we found the following the values for the cross section at the ω and ϕ peaks:

$$\begin{aligned} \sigma^{e^+e^- \rightarrow \pi\gamma}(m_\omega^2) &= 177 \text{ nb}, \\ \sigma^{e^+e^- \rightarrow \pi\gamma}(m_\phi^2) &= 5.5 \text{ nb}. \end{aligned} \quad (13)$$

Predictions for the value of the cross section of annihilation into the $\pi(1300)\gamma$ pair are given in fig. 5. The peak due to ρ' and ω' meson resonances is clearly seen there, but

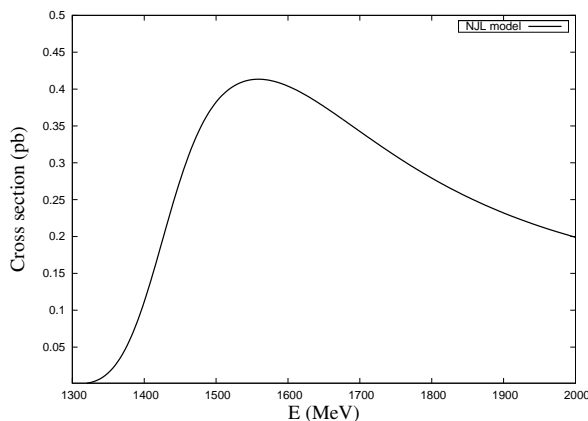


Figure 5. NJL model prediction for the cross section of $e^+e^- \rightarrow \pi^0\gamma$ process.

the magnitude of the peak is small. Partially that is due to mutual compensation of the ground and excited states in $V_{\rho'\pi'\gamma}$. Note that the NJL model is adjusted for applications at low energies up to about 2 GeV. In this energy range, the model gives a qualitative description of meson properties and interactions. The advantage is that the set of parameters is limited and fixed. It is worth to stress that to describe the given processes we did not introduce any new parameter in the model.

4 Conclusions

Our calculation for the process $e^+e^- \rightarrow \pi^0\gamma$ showed the presence of two resonance regions in the energy range below 1.1 GeV. The first resonance appears in the region of the ω meson mass and looks as a very high narrow peak. The second one is also narrow, it lies in the region of ϕ meson mass. The dip just after the ϕ meson peak is due to destructive interference of amplitudes. At the present time this process is studied with high accuracy only in the energy region below 1 GeV [8]. It is worth to note that the experimental energy dependence of the process cross section in the first both resonance region is very close to our theoretical estimation. Ongoing experiments at VEPP-2000 and BES-III collect statistics at higher energies. This will allow us to perform a comparison of our theoretical predictions with future experimental data. The cross section of $\pi^0\gamma$ production has the order of about 1 pb, so it is difficult to measure.

Further we are going to consider radiative processes with participation of η , η' , ρ , ω , ϕ mesons and their radial excited states in the framework of the extended $U(3) \times U(3)$ NJL model. We plan to consider also production of these particles in e^+e^- collisions.

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